physical quantities (the term 'relation' being used instead of speaking about 'groups of groups'); e.g., one would find that

$$X_1 X_3^2 X_5^{-1} = 1$$

constitutes a simple relation. Once again, one could write down the set of all simple relations and continue with the hierarchy of groups, relations, and so on. However, this procedure seems to be a delicacy for the mathematically interested reader only.

Finally, a remark about the boundaries of dimensional analysis is due. Taking a *triplet* of linearly independent groups, see equation (18), e.g. $\{X_1, X_2, X_3\}$, by dimensional analysis we only know that there is a functional relationship between the groups considered, say

$$f(X_1, X_2, X_3) = 0.$$

Only further investigations (experimental or theoretical) can give more information about the actual form of this relationship.

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APPENDIX: EXAMPLE FOR A GROUP THAT IS NOT SIMPLE

A nonsimple group might be characterised in such a way that it can always be 'simplified', i.e. roughly speaking, by omitting some of the physical quantities occurring in it, the rest will constitute a simple group; e.g., in the former example, a nonsimple group is the following:

$$X = A_1^2 A_2^{-3} A_4^{-1} A_5 A_6.$$

Check: $\mathbf{x} = [2, -3, 0, -1, 1, 1,]^T$ is in fact a particular solution of equation (4) with (17).

X can be simplified in several ways:

(1) Deleting A_1, the rest constitutes 'Navier'.
(2) Deleting A_2, the rest constitutes 'd'Alembert'.
(3) Deleting A_5, the rest constitutes 'Poiseuille'.
(4) Deleting A_4 and A_6, the rest constitutes 'Froude'.

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Temperature ratio effects in compressible turbulent boundary layers

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1. INTRODUCTION

THIS PAPER describes an experimental and theoretical investigation of the heat transfer coefficient for a compressible, constant pressure, turbulent boundary layer on an isothermal flat plate. In the presence of large temperature differences between the freestream and the wall, the Nusselt number can be expected to depend on the ratio $T_w/T_{t\infty}$, due to compressibility effects and variations in gas properties with temperature through the boundary layer. At constant freestream Reynolds number, this is generally written in the form

$$Nu = Nu_i(T_w/T_{t\infty})^n.$$

There is a lack of experimental data in the literature, but reported analytical work for air tends to suggest a decrease in Nu with wall-to-gas temperature ratio. Kays and Crawford [1], for example, give a value for n of -0.4 for $T_w/T_{t\infty} > 1$; Eckert's reference temperature [2] corresponds to an exponent of -0.19 for the conditions investigated in this paper. Brown's computations of flat plate heat transfer [3] also show a decrease of Nu with $T_w/T_{t\infty}$, as do the turbulent heat transfer charts presented by Neal and Bertram [4]. Bose [5] solves the turbulent boundary-layer equations numerically for $0.1 < T_w/T_{t\infty} < 0.9$ and lists different St–Re correlations for the three temperature ratios which he considers. In all the cases described above, a negative value for the exponent n could be inferred. Previous experimental work at Oxford by Loftus and Jones [6] suggested that this effect was small.

This paper examines the mechanisms for the dependence of Nusselt number on wall-to-gas temperature ratio. In addition, experimental results are presented for air at $M = 0.55$ and $Re/m = 2.7 \times 10^7\ \mathrm{m}^{-1}$, for $0.5 < T_w/T_{t\infty} < 1.3$. This data is compared with numerical solutions of the turbulent, compressible boundary-layer equations using conventional mixing length turbulence models.

2. ANALYTICAL DISCUSSION OF THE TEMPERATURE RATIO EFFECT

Although the effect of the wall-to-gas temperature ratio is complicated, some understanding of possible mechanisms for producing such changes in the Nusselt number can be gained from a study of the laminar compressible boundary-layer equations, which for convenience can be considered in the simplified form

$$\left.\begin{aligned} &ff'' + (Cf'')' = 0 \\ &[(C/Pr)T']' + fT' + (\gamma-1)M_\infty^2 C T_\infty f''^2 = 0 \end{aligned}\right\} \quad (1)$$

with boundary conditions

$$f(0) = f'(0) = 0, \quad T(0) = T_w, \quad f'(\infty) = 1, \quad T(\infty) = T_\infty$$

where

$$' = \frac{d}{ds}, \quad T = T(s), \quad \Psi = \sqrt{(2\mu_\infty U_\infty x/\rho_\infty)} f(s),$$
$$C = \mu\rho/\mu_\infty\rho_\infty$$

NOMENCLATURE

C_p specific heat at constant pressure
k thermal conductivity [$W\,m^{-1}\,K^{-1}$]
q heat transfer [$W\,m^{-2}$]
T temperature [K]
U velocity [$m\,s^{-1}$]
x distance down plate [m].

Greek symbols
μ dynamic viscosity [$kg\,m^{-1}\,s^{-1}$]
ν kinematic viscosity [$m^2\,s^{-1}$]
ρ density [$kg\,m^{-3}$]
Ψ streamfunction [$m^2\,s^{-1}$].

Non-dimensional parameters
α $U_\infty^2/(c_p T_\infty)$
C $\mu\rho/\mu_\infty\rho_\infty$
θ T/T_∞
M Mach number
Nu Nusselt number, $q/[k_{t\infty}(T_{t\infty}-T_w)]$
Pr Prandtl number, $\mu C_p/k$
Re Reynolds number, xU_∞/ν
St Stanton number, $Nu/(Pr\,Re)$
γ ratio of specific heats.

Subscripts
i incompressible constant property
w evaluated at the wall
t∞ evaluated at total conditions in the freestream.

and the variable s is defined by

$$s = \sqrt{[\rho_\infty U_\infty/(2x\mu_\infty)]}\int_0^y \frac{\rho}{\rho_\infty}\,dy.$$

It is easy to show from the above that for $\mu = KT$, where K is a constant, so that viscosity is assumed proportional to temperature, and arbitrary Prandtl number, the solution to the temperature equation is

$$T/T_\infty = 1+Pr(\gamma-1)M_\infty^2\int_\eta^\infty\left\{[f''(t)]^{Pr}\int_0^t [f''(q)]^{2-Pr}\,dq\right\}dt + B\int_\eta^\infty [f''(t)]^{Pr}\,dt$$

where B is a constant. For $Pr = 1$, the Nusselt number is given by

$$Nu = \frac{\sqrt{(Re/2)}C_w T'(0)}{T_\infty[1+\alpha/2-T_w/T_\infty]} = \sqrt{(Re/2)}f''(0)C_w. \quad (2)$$

The momentum equation is merely the Blasius equation, so that in this case, there is no temperature ratio effect on the Nusselt number.

If it is now assumed that $C = g(T)$, where g is arbitrary, so that more complicated temperature–viscosity laws can be considered, the temperature equation has the solution

$$T = T_\infty + \tfrac{1}{2}\alpha T_\infty(f'-f'^2)+(T_w-T_\infty)(1-f').$$

(assuming $Pr = 1$, although this analysis can be carried out if this is not the case). The expression for the Nusselt number is once again (2). However, the momentum equation satisfied by f is now no longer that of Blasius. If, for example, the viscosity law $\mu \propto T^m$ is used for some $\alpha < 1$, it can be seen that

$$C = \left(\frac{T}{T_\infty}\right)^{m-1}$$

so that the momentum equation becomes

$$ff''+\{[1+\tfrac{1}{2}\alpha(f'-f'^2)+(T_w/T_\infty-1)(1-f')]^{m-1}f''\}' = 0 \quad (3)$$

with boundary conditions as in (1). This enables $f''(0)$ to be found by a regular perturbation analysis when θ is close to unity so that (2) becomes

$$Nu = 0.4946(T_w/T_\infty)^{0.2411(m-1)}\sqrt{(Re/2)}. \quad (4)$$

When the flow is turbulent, analysis is clearly far more complicated, but it seems plausible that the temperature ratio effect will be produced by a mechanism similar to that described above.

3. NUMERICAL INVESTIGATION

In order to help understanding of the experimental results, a computer program was written in FORTRAN 77 to analyse the flow field numerically. The code was based on the non-similar, steady, compressible, turbulent boundary-layer programs of Cebeci [7], Cebeci and Smith [8] and Blottner [9], and applied to a flat plate flow with a constant freestream velocity. It was also assumed that the wall temperature was specified and constant.

In order to produce a more computationally convenient set of equations, the boundary-layer equations were first transformed using Lees–Levy variables (see, for example Lees [10] for an explanation of this transformation) to give

$$\begin{aligned} &2\xi F_\xi + V_\eta + F = 0 \\ &2\xi FF_\xi + VF_\eta - (LF_\eta)_\eta = 0 \\ &2\xi F\theta_\xi + V\theta_\eta - (N\theta_\eta)_\eta = \alpha(L-M)F_\eta^2 - \alpha F(MF_\eta)_\eta \end{aligned} \quad (5)$$

with boundary conditions

$$\eta = 0:\ F = V = 0,\ \ \theta = T_w/T_\infty$$
$$\eta \to \infty:\ F = \theta = 1$$

where

$$F = u/U_\infty,\quad V = -\sqrt{(2\xi)}\Psi_\xi,\quad \theta = T/T_\infty,$$
$$L = C(1+\varepsilon_m/\nu),\quad N = C(1/Pr+\varepsilon_h/\nu),\quad M = (C/\nu)(\varepsilon_m-\varepsilon_h)$$
$$\xi = \int_0^x U_\infty\rho_\infty\mu_\infty\,dx,\quad \eta = \frac{(\rho U)_\infty}{\sqrt{(2\xi)}}\int_0^y \frac{\rho}{\rho_\infty}\,dy.$$

This parabolic system of second-order equations was discretized directly using a Crank–Nicholson implicit scheme. In order to use the linearity of the temperature equation to its full advantage, the continuity and momentum equations were solved iteratively before the temperature equation, the cycle then being repeated until convergence was achieved. Since it was not envisaged that the program would be run at extreme gas temperatures, constant specific heats were used, and the viscosity was given by Sutherland's law or a user-specified function. The Prandtl number variation with temperature was taken to be $0.72\,[1-2.92\times10^{-4}(T-250)]$, obtained from data in [11]. The turbulence model used was similar to that given by Bose [5] which is based on a mixing length theory and employs the concept of a turbulent Prandtl number to relate the eddy viscosity and eddy conductivity. Changing the turbulent Prandtl number model did not significantly affect the results.

A comparison between laminar numerical calculations, using a viscosity–temperature relationship $\mu \propto T^m$ for various values of m, and the analytical prediction (4) is given in Fig. 1.

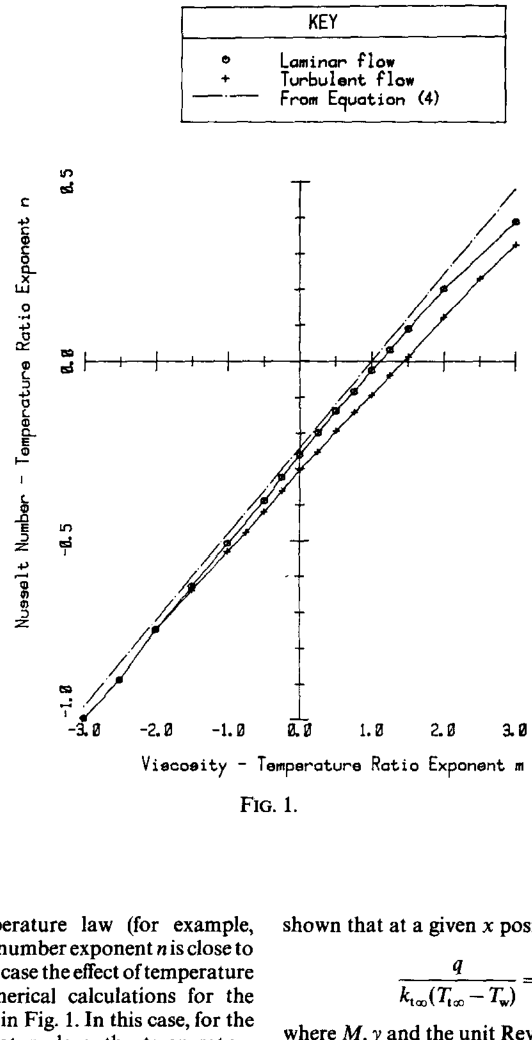

FIG. 1.

For a realistic viscosity–temperature law (for example, $\mu \propto T^{0.76}$), the resulting Nusselt number exponent n is close to zero, showing that in the laminar case the effect of temperature ratio is small. Equivalent numerical calculations for the turbulent flow are also included in Fig. 1. In this case, for the same realistic viscosity–temperature law, the temperature ratio dependence is more pronounced.

4. EXPERIMENTAL INVESTIGATION

Experimental results were obtained for air flowing over a flat plate with zero pressure gradient using an Isentropic Light Piston Tunnel. This is a transient facility in which test gas is compressed in a tube by an air-driven piston before flowing through the working section at constant temperature and pressure. A comprehensive description of its operation can be found in [12] and [6].

The Reynolds number per metre was 2.7×10^7, and $M = 0.55$. Three gas total temperatures, 280, 365 and 500 K, were attained, the first by pre-cooling the test gas and the other two by changing the compression ratio in the pump tube. It was possible to pre-heat or pre-cool the flat plate by forced convection to give a range of wall temperatures $300 < T_w < 365$ K and thus an overall temperature ratio $0.60 < T_w/T_{t\infty} < 1.30$. The boundary layer was tripped at $Re = 5.4 \times 10^5$.

Wall temperature and spanwise averaged heat transfer rates were measured using thin film gauges, the theory and use of which is described in [13]. Gas temperatures were measured using a fast response micro-thermocouple mounted in a stagnation probe. Data was acquired and processed on a PDP 11/10 mini-computer.

For a compressible flow with constant C_p, constant Pr and a power law $\mu \propto T^m$ relating viscosity to temperature, it can be shown that at a given x position,

$$\frac{q}{k_{t\infty}(T_{t\infty} - T_w)} = \text{constant}\, f(T_w/T_{t\infty}) \tag{6}$$

where M, γ and the unit Reynolds number are constant. This can be expressed as

$$q/k_{t\infty}T_{t\infty} = \text{constant}\,(1 - T_w/T_{t\infty})f(T_w/T_{t\infty}). \tag{7}$$

If $f(T_w/T_{t\infty}) = 1$, $q/k_{t\infty}T_{t\infty}$ would vary linearly with $T_w/T_{t\infty}$; if not, any curvature could be approximated by a power law

$$f(T_w/T_{t\infty}) = (T_w/T_{t\infty})^n$$

so that $Nu = Nu_i(T_w/T_{t\infty})^n$.

Figures 2(A)–(D) show experimental data presented in the manner of (7), where $q/k_{t\infty}T_{t\infty}$ is plotted against $T_w/T_{t\infty}$. This is compared with numerical results given by the solid lines for four positions on the flat plate. Errors due to non-isothermal wall temperature distributions have been evaluated using the method described in [6] and were found to be typically of the order of 1%. The data clearly exhibits curvature, and representing the $T_w/T_{t\infty}$ effect by a power law over this range, the experimental value of n is between -0.21 and -0.28 for the x positions where measurements were taken. This shows no systematic variation with distance. Numerical calculations produce similar trends although the curvature is slightly less, with n ranging from -0.13 to -0.18 and increasing in magnitude with distance.

5. CONCLUSIONS

It has been shown numerically and experimentally that the Nusselt number for a turbulent boundary layer exhibits a dependence on the wall-to-gas temperature ratio. Analysis of the laminar case reveals how this dependence relates to the

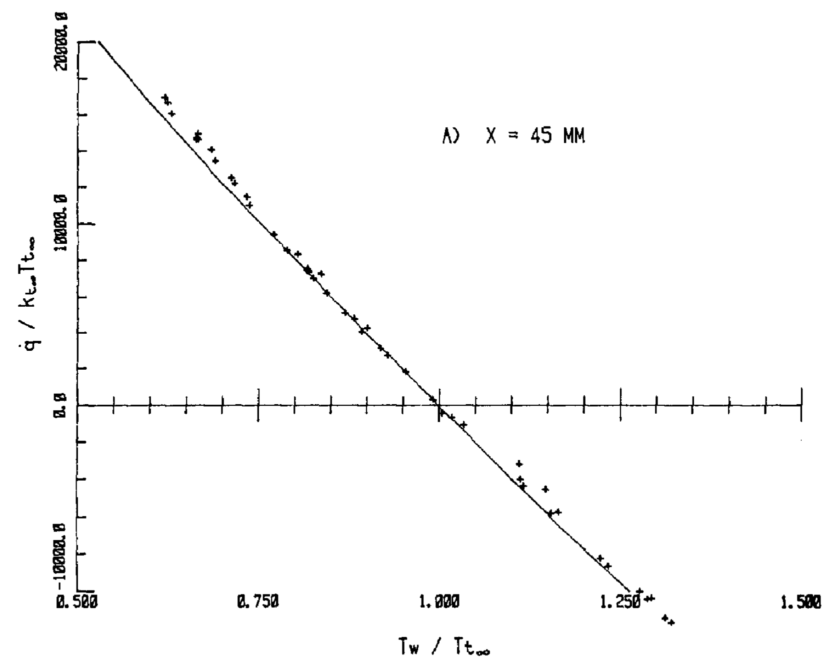
A) X = 45 MM
20000.0
10000.0
0.0
-10000.0
0.500
0.750
1.000
1.250
1.500
q̇ / k$_{t\infty}$Tt$_\infty$
Tw / Tt$_\infty$

FIG. 2(A).

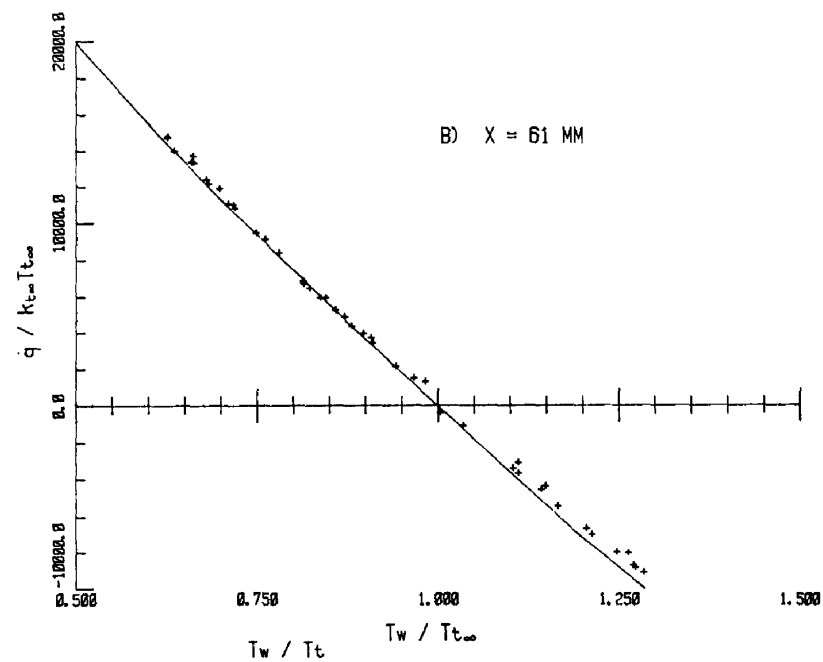
B) X = 61 MM
20000.0
10000.0
0.0
-10000.0
0.500
0.750
1.000
1.250
1.500
q̇ / k$_{t\infty}$Tt$_\infty$
Tw / Tt$_\infty$
Tw / Tt

FIG. 2(B).

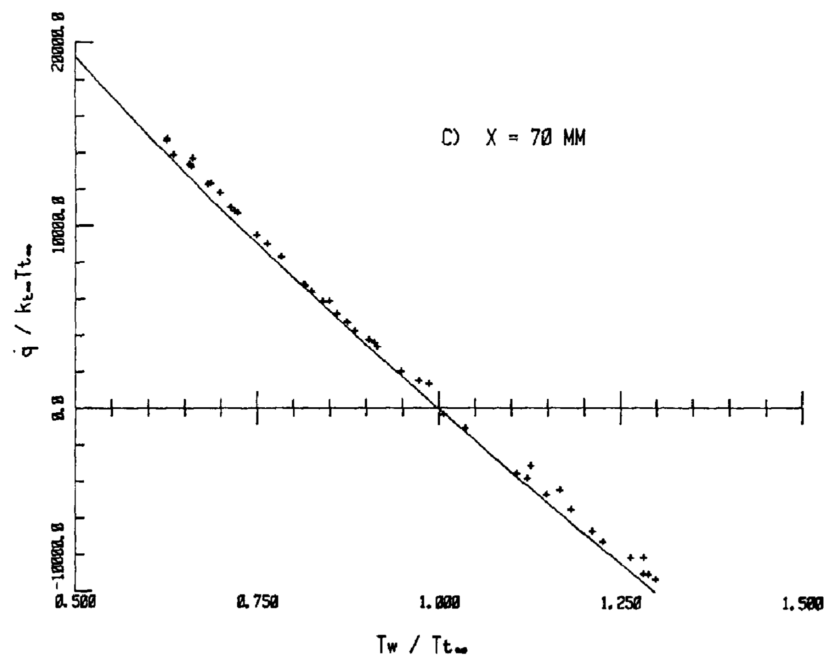
C) X = 70 MM
20000.0
10000.0
0.0
-10000.0
0.500
0.750
1.000
1.250
1.500
q / kt∞ Tt∞
Tw / Tt∞

FIG. 2(C).

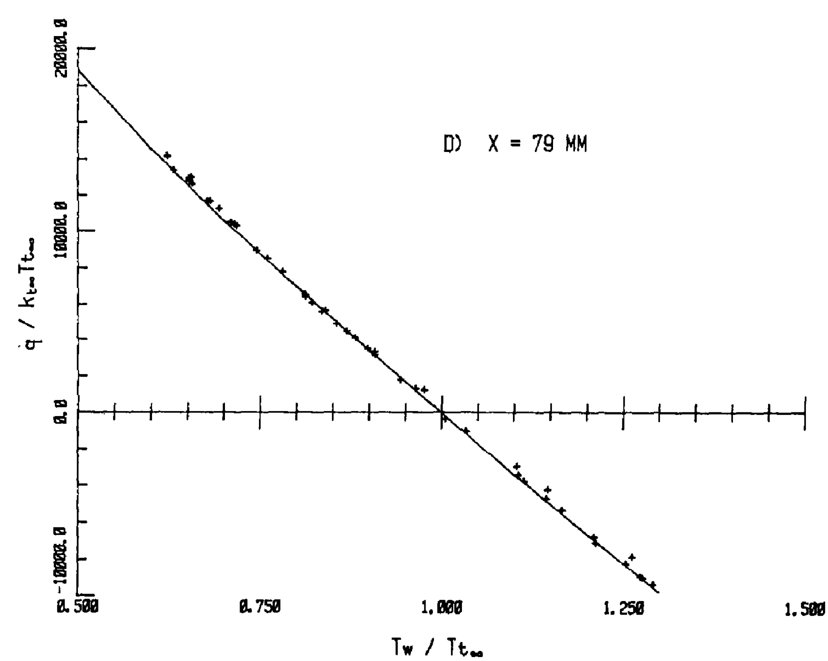
D) X = 79 MM
20000.0
10000.0
0.0
-10000.0
0.500
0.750
1.000
1.250
1.500
q / kt∞ Tt∞
Tw / Tt∞

FIG. 2(D).

variation of ρ and transport properties with temperature through the boundary layer.

The authors suggest, on the basis of experimental data, the following turbulent boundary-layer relations for air in the range $0.6 < T_w/T_{t\infty} < 1.3$:

$$Nu = Nu_i(T_w/T_{t\infty})^{-0.25}. \quad (8)$$

Outside this range, or for other gases, a numerical solution of the turbulent boundary-layer equations as described in this paper will give a good indication of the dependence of the Nusselt number on wall-to-gas temperature ratios.

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Effect of suction on heat transfer rates from a rotating cone

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INTRODUCTION

INVESTIGATIONS on rotating systems have attained immense technological importance and, in that context, heat transfer systems from various types of axisymmetric bodies are presented in the literature [1]. Kreith [2] investigated the problem of cones and disks in turbulent and mixed flow. Sparrow and Gregg [3] analysed theoretically the problem of laminar heat transfer from a rotating disk with suction applied to the wall. Their analysis was restricted purely to the forced convective conditions and buoyancy effects were not considered. Hartnett and Deland [4] solved the problem of forced convection from rotating non-isothermal disks and cones with the intention of studying the Prandtl number effects on heat transfer rates. Herring and Grosh [5] studied heat transfer rates from a cone to air with the inclusion of buoyancy forces. Bergles [6, 7] classified rotation of the heat transfer surface as an active augmentation technique. The present investigation deals with a compound technique, namely rotation with simultaneous application of suction at the surface of a right vertical inverted cone, to assess the degree of augmentation achieved by treating the problem in its general form from which the special cases [1, 4, 5] can be arrived at.

FORMULATION OF THE PROBLEM

The configuration and disposition of the rotating cone with the coordinate system is the same as that given in ref. [5] save for application of suction of a constant value at the surface of the cone. The dimensionless boundary-layer equations for a steady, non-dissipative, constant property, axisymmetric flow are as follows:

Law of continuity:

$$2F + H' = 0. \quad (1)$$

Conservation of momentum:
x-direction (meridional)

$$F'' - (H - \beta_s)F' - F^2 + G^2 + \left(\frac{Gr}{Re^2}\right)\theta = 0 \quad (2)$$

y-direction (tangential)

$$G'' - (H - \beta_s)G' - 2FG = 0. \quad (3)$$

Energy equation:

$$\theta'' - Pr[(H - \beta_s)\theta' + F\theta] = 0. \quad (4)$$

The following velocity, temperature and dimensionless spatial functions are employed to arrive at the similarity transformations, i.e. equations (1)–(4):

$$u = x\omega \sin \alpha F(\eta) \quad (5)$$

$$v = x\omega \sin \alpha G(\eta) \quad (6)$$

$$w = (\nu\omega \sin \alpha)^{1/2}[H(\eta) - \beta_s] \quad (7)$$

$$(T - T_\infty) = (T_w - T_\infty)\theta(\eta) \quad (8)$$

$$\eta = (\omega \sin \alpha/\nu)^{1/2} z \quad (9)$$

$$Gr = g \cos \alpha (T_w - T_\infty)\beta x^3/\nu^2 \quad (10)$$

$$Re = \omega \cos \alpha x^2/\nu. \quad (11)$$

When $\beta_s = 0$, equations (1)–(4) would be identical to those solved by Herring and Grosh [5]. The above equations are to